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Transducer design, Part 1
MEDT8007 winter 2010

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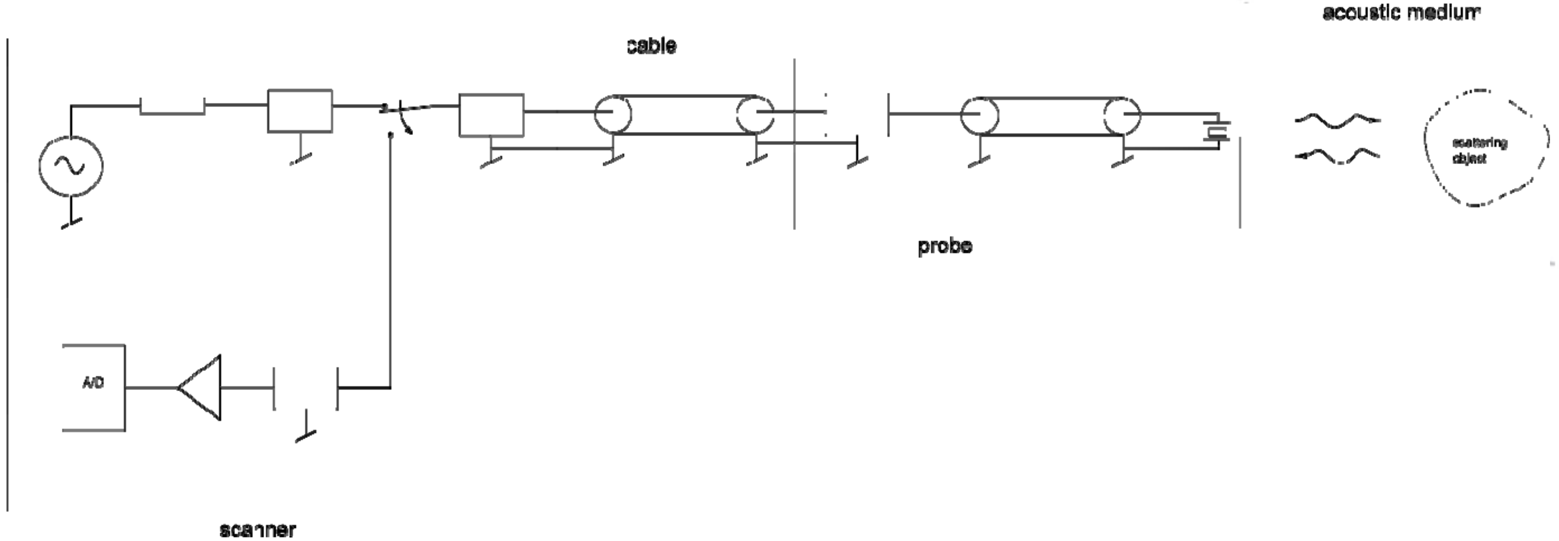
Overview, ultrasound signal path, single channel

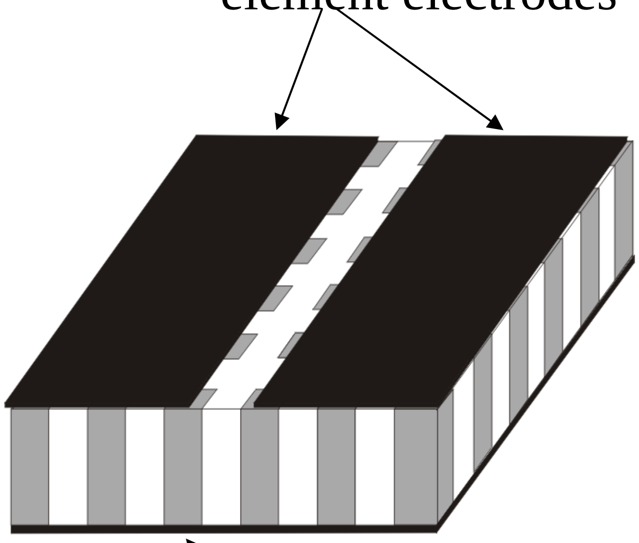


Scanner components:
- Transmit section
- Preamplifier
- A/D converter
- TR switch (expander/limiter)
- Others

Cable components:
- Coaxial cables
- Connectors
- Tuning electronics

Probe components:
- Transducer
- Tuning electronics
- Internal wiring
- Electronics

Overview, ultrasound transducer array piezo composite.



Piezo ceramic – diced filled with polymer

New ”equivalent material”

- •better mechanical matching
- •geometrical shaping
- •less lateral coupling

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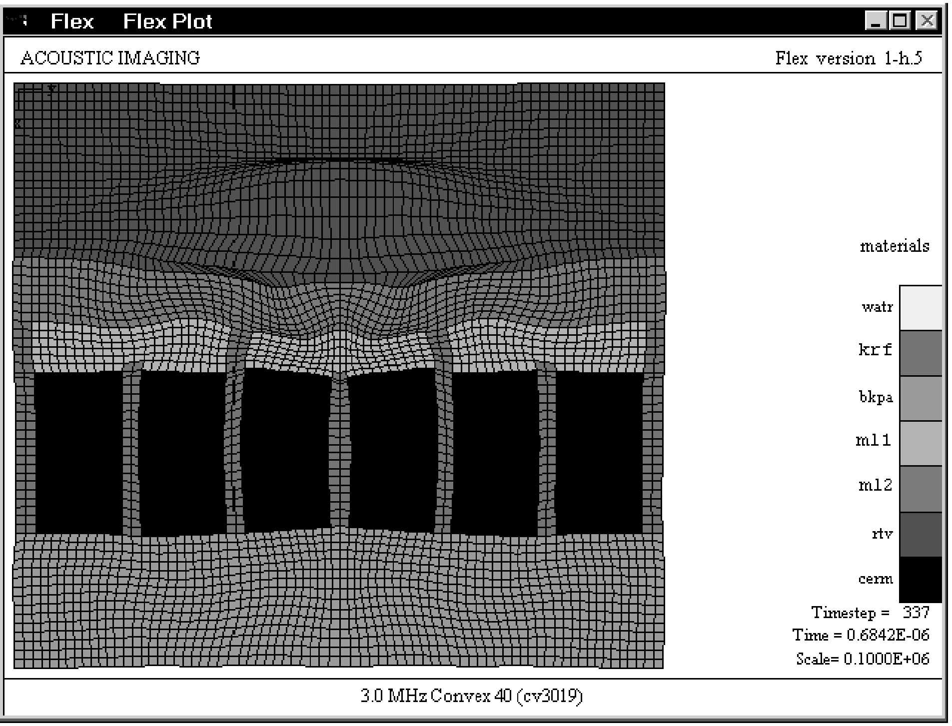
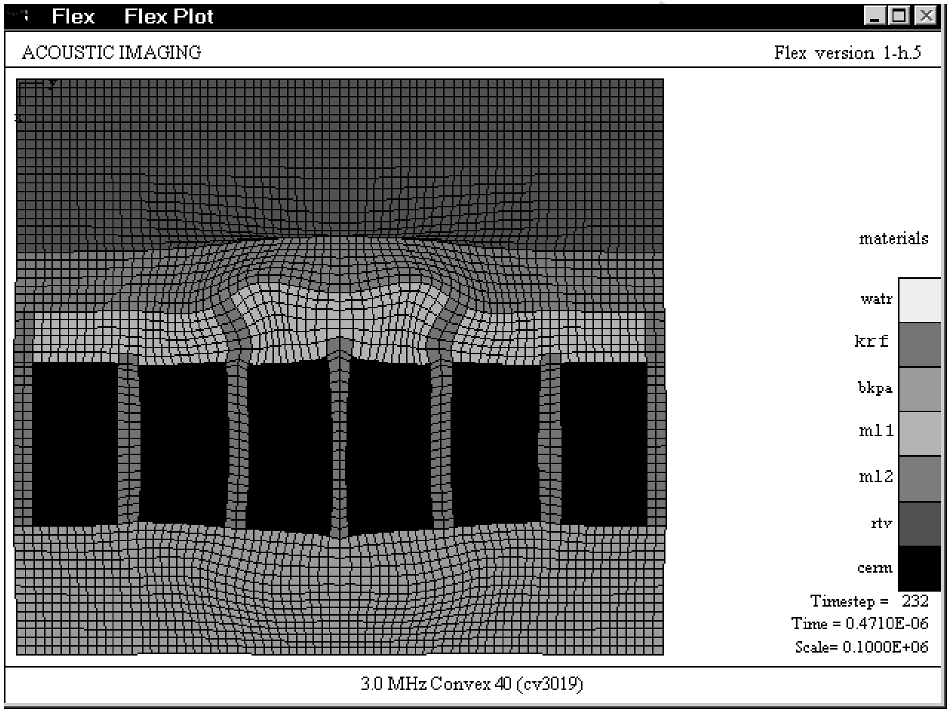
Overview, ultrasound transducer array piezo composite, cntd.





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Overview

- Plane wave propagation
  - Mechanical waves
  - Electro magnetic waves in coaxial cable
  - Transfer matrix
- Piezo electric materials
  - Constitutive equations
  - Plane wave model (Mason)
  - Signal model

Litterature:

B.A.J.Angelsen, Ultrasound Imaging , vol.1 ch.2, 3
R.S.C. Cobbold, Foundations of Biomedical Ultrasound, ch. 1.5, ch.6
McKeighen, R.E., Design guidelines for medical ultrasonic arrays, pp. 2-18 in: Ultrasonic Transducer Engineering, K.K. Shung, ed., Proc. SPIE vol. 3341, Medical Imaging, 1998.

Plane waves

P_+, U_+ →

P_-, U_- ←

Acoustical/mechanical

$$p(z,t) = \left(P_+ e^{-jkz} + P_- e^{jkz}\right) e^{j\omega t}$$
$$u(z,t) = \left(U_+ e^{-jkz} + U_- e^{jkz}\right) e^{j\omega t}$$

$$P_+ = Z_1 U_+ \quad , \quad P_- = -Z_1 U_-$$

$$Z_1 = \rho c \quad , \quad c = \sqrt{\frac{1}{\rho c}} \quad , \quad k = \frac{\omega}{c}$$

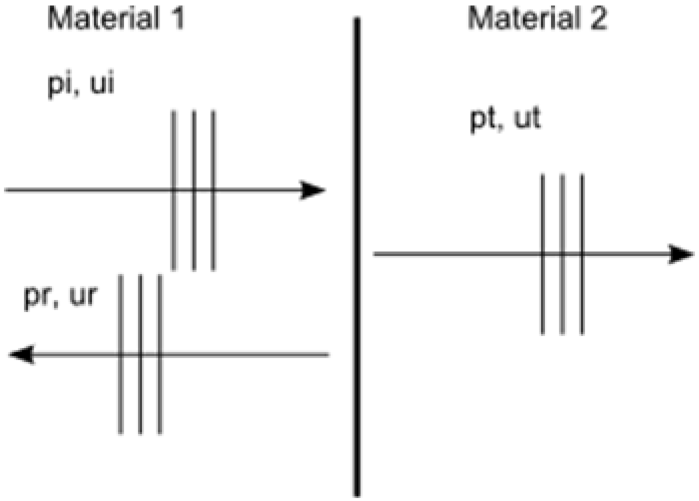
Electrical

$$V(z,t) = \left(V_+ e^{-jkz} + V_- e^{jkz}\right) e^{j\omega t}$$
$$I(z,t) = \left(I_+ e^{-jkz} + I_- e^{jkz}\right) e^{j\omega t}$$

$$V_+ = Z_1 I_+ \quad , \quad V_- = -Z_1 I_-$$

$$Z_1 = \sqrt{\frac{L}{C}} \quad , \quad c = \sqrt{\frac{1}{LC}} \quad , \quad k = \frac{\omega}{c}$$

Interface between two materials



Continuity in pressure and velocity at boundary:

$p_i + p_r = p_t$, $u_i + u_r = u_t$

Wave amplitude relations

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

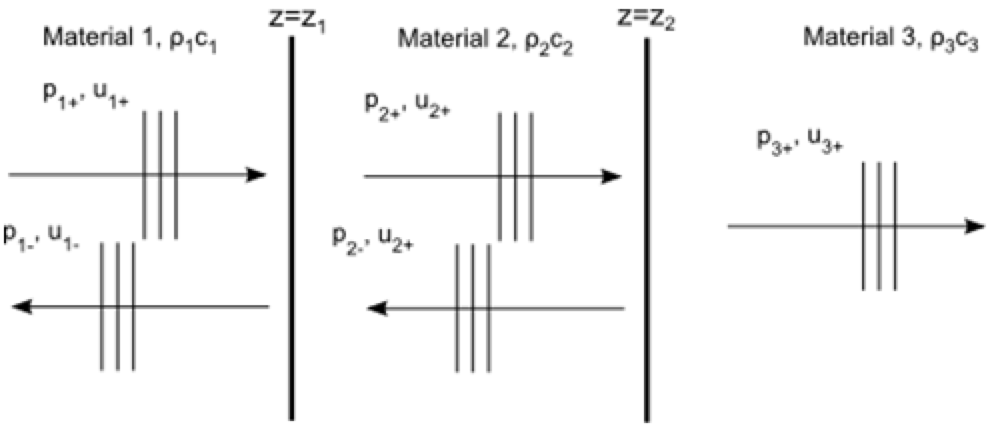
$$T = \frac{2Z_2}{Z_2 + Z_1}$$

Wave energy relations

$$r = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

$$\tau = 1 - r = \frac{4Z_1 Z_2}{(Z_2 + Z_1)^2}$$

Interface between three materials



Continuity in pressure and velocity at boundaries:

$$\left.\begin{aligned} p_{1+} + p_{1-} &= p_{2+} + p_{2-} \\ u_{1+} + u_{1-} &= u_{2+} + u_{2-} \end{aligned}\right\} \text{for } z = z_1$$

$$\left.\begin{aligned} p_{2+} + p_{2-} &= p_{3+} \\ u_{2+} + u_{2-} &= u_{3+} \end{aligned}\right\} \text{for } z = z_2$$

Impedance into Material 2

$$Z_{i2} = \rho_2 c_2 \frac{\rho_3 c_3 + j\rho_2 c_2 \tan\left(k_2 L_2\right)}{\rho_2 c_2 + j\rho_3 c_3 \tan\left(k_2 L_2\right)}$$

$$L_2 = z_2 - z_1$$

$$Z_{i2} = \frac{\left(\rho_2 c_2\right)^2}{\rho_3 c_3}, \text{ for } k_2 L_2 = \frac{\pi}{2} \Rightarrow L_2 = \frac{\lambda_2}{4}$$

$$Z_{i2} = \rho_3 c_3 \qquad , \text{ for } k_2 L_2 = \pi \Rightarrow L_2 = \frac{\lambda_2}{2}$$

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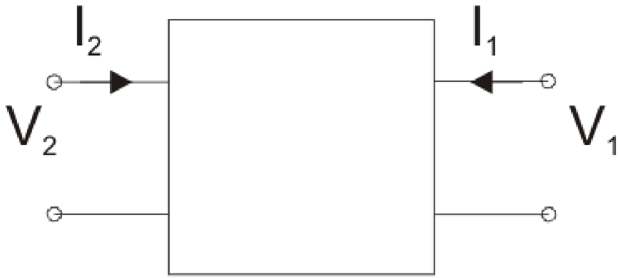
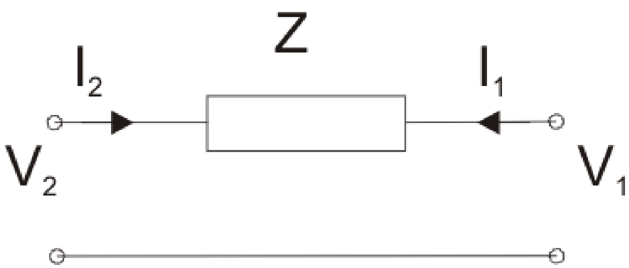
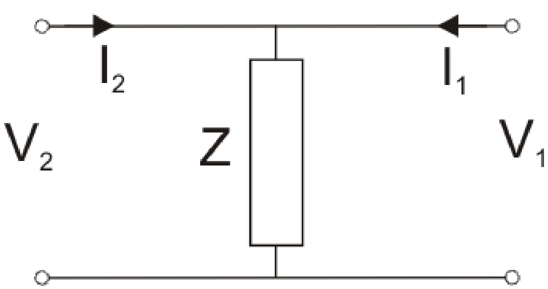
Transfer matrix – simple results



$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = A \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

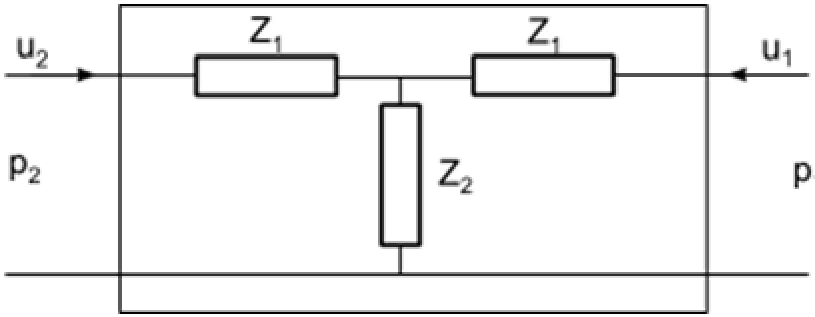


$$\begin{aligned} I_2 &= -I_1 \\ V_2 &= V_1 - ZI_1 \end{aligned} \quad \Rightarrow \quad A = \begin{bmatrix} 1 & -Z \\ 0 & -1 \end{bmatrix}$$



$$\begin{aligned} I_2 &= -\left(I_1 - \frac{V_1}{Z} \right) \\ V_2 &= V_1 \end{aligned} \quad \Rightarrow \quad A = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & -1 \end{bmatrix}$$

2 port model, 1D layer (plane wave)



Transfer matrix

$$\begin{pmatrix} p_2 \\ u_2 \end{pmatrix} = \begin{pmatrix} \cos kL & -jZ_1 \sin kL \\ \frac{j\sin kL}{Z_1} & -\cos kL \end{pmatrix} \begin{pmatrix} p_1 \\ u_1 \end{pmatrix}$$

$$Z_1 = j\rho c \tan\frac{kL}{2}$$

$$Z_2 = -j\frac{\rho c}{\sin kL}$$

Evaluation in layer m:

$$p_{m,2} = p_{m,+} e^{-jk_m z_{m,2}} + p_{m,-} e^{+jk_m z_{m,2}}$$

$$p_{m,1} = p_{m,+} e^{-jk_m z_{m,1}} + p_{m,-} e^{+jk_m z_{m,1}}$$

etc.

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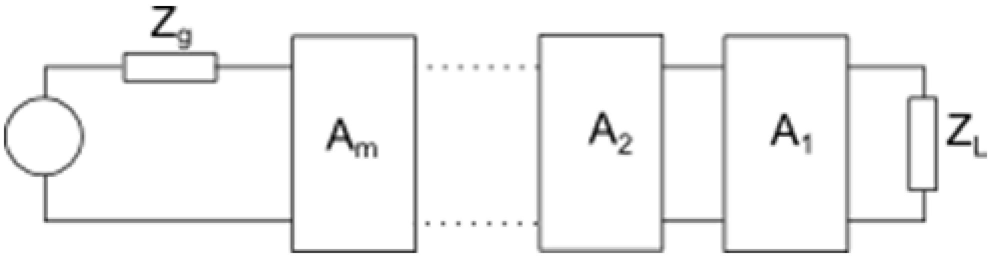
A simple algorithm



1. Compute impedance as seen into A_m from Z_g
   1. Propagate Z_L through chain of tr.matrices
2. Compute $p_{m,2}$ and $u_{m,2}$ at the source
3. Propagate p and u through chain with tr.matrices.

$$p_{1,1} = -u_{1,1} Z_L$$

$$\begin{pmatrix} p_{1,2} \\ u_{1,2} \end{pmatrix} = A_1 \begin{pmatrix} p_{1,1} \\ \dfrac{-p_{1,1}}{Z_L} \end{pmatrix} = \begin{pmatrix} a_{1,11} p_{1,1} - \dfrac{a_{1,12}}{Z_L} p_{1,1} \\ a_{1,21} p_{1,1} - \dfrac{a_{1,22}}{Z_L} p_{1,1} \end{pmatrix}$$

$$Z_{i,1} = \frac{a_{1,11} - \dfrac{a_{1,12}}{Z_L}}{a_{1,21} - \dfrac{a_{1,22}}{Z_L}} = Z_{L,2}$$

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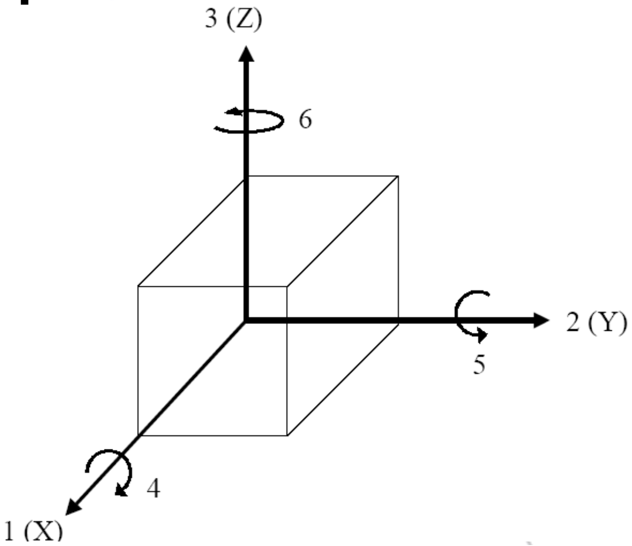
Piezoelectric materials, 3D properties



$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} c_{11}^D & c_{12}^D & c_{13}^D & 0 & 0 & 0 \\ c_{12}^D & c_{22}^D & c_{13}^D & 0 & 0 & 0 \\ c_{13}^D & c_{13}^D & c_{33}^D & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^D & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}^D & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^D \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} - \begin{pmatrix} 0 & 0 & h_{13} \\ 0 & 0 & h_{13} \\ 0 & 0 & h_{33} \\ 0 & h_{15} & 0 \\ h_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}$$

c_{ij}^D stiffness tensor

h_{ij} p.e.strain coeff.

ε_{ij}^S dielectric perm.

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = - \begin{pmatrix} 0 & 0 & 0 & 0 & h_{15} & 0 \\ 0 & 0 & 0 & h_{15} & 0 & 0 \\ h_{13} & h_{13} & h_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} + \begin{pmatrix} \frac{1}{\varepsilon_{11}^S} & 0 & 0 \\ 0 & \frac{1}{\varepsilon_{22}^S} & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{33}^S} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}$$

T – tension

S – strain

D – elec.displacement

E – elec.field

Piezoelectric material 1D material equations

$$T_3 = c_{33}^D S_3 - h_{33} D_3$$

$$E_3 = -h_{33} S_3 + \frac{1}{\varepsilon_{33}^S} D_3$$

$$p(z,t) = -\frac{1}{\kappa}\frac{\partial \psi(z,t)}{\partial z} + hD(z,t)$$

$$E(z,t) = -h\frac{\partial \psi(z,t)}{\partial z} + \frac{1}{\varepsilon} D(z,t)$$

$$\mathrm{V(t)} = \int_0^L \mathrm{E(z,t)dz}$$

$$= -\mathrm{h}\left\{\psi(\mathrm{L,t}) - \psi(0,\mathrm{t})\right\} + \frac{1}{\mathrm{C_0}}\mathrm{q(t)}$$

$$\mathrm{V}(\omega) = \frac{1}{\mathrm{j}\omega \mathrm{C_0}}\mathrm{I}(\omega) + \frac{\mathrm{h}\left(\mathrm{U(L},\omega) - \mathrm{U}(0,\omega)\right)}{\mathrm{j}\omega}$$

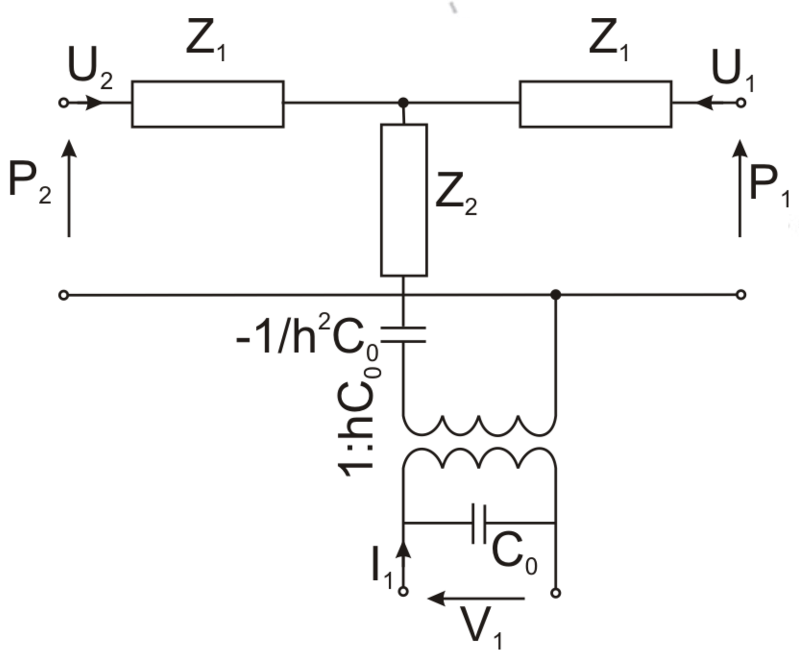
Wave equation

$$\left.\begin{aligned} &\kappa\frac{\partial \mathrm{p(z,t)}}{\partial \mathrm{t}} - \frac{\partial \mathrm{u(z,t)}}{\partial \mathrm{z}} = \kappa\frac{\partial \mathrm{hq(t)}}{\partial \mathrm{t}} \\ &\rho\frac{\partial \mathrm{u(z,t)}}{\partial \mathrm{t}} - \frac{\partial \mathrm{p(z,t)}}{\partial \mathrm{z}} = 0 \end{aligned}\right\} \Rightarrow \frac{\partial^2 \mathrm{p(z,t)}}{\partial \mathrm{z}^2} - \frac{1}{\mathrm{c}^2}\frac{\partial^2 \mathrm{p(z,t)}}{\partial \mathrm{t}^2} = \kappa\frac{\partial^2 \mathrm{hq(t)}}{\partial \mathrm{t}^2}$$

$$\mathrm{c} = \sqrt{\frac{1}{\rho\kappa}} \qquad \mathrm{Z_0} = \rho \mathrm{c} = \sqrt{\frac{\rho}{\kappa}}$$

Mason electrical equivalent model

- 3 port model, (2 mechanical, 1electrical)
- transmission line describe vibration
- mechanical and electrical variables can be extracted from the material equations



$$Z_1 = iZ_0 \tan\frac{kL}{2} \quad , \quad C_0 = \varepsilon \frac{A}{t}$$

$$Z_2 = \frac{iZ_0}{\sin kL}$$

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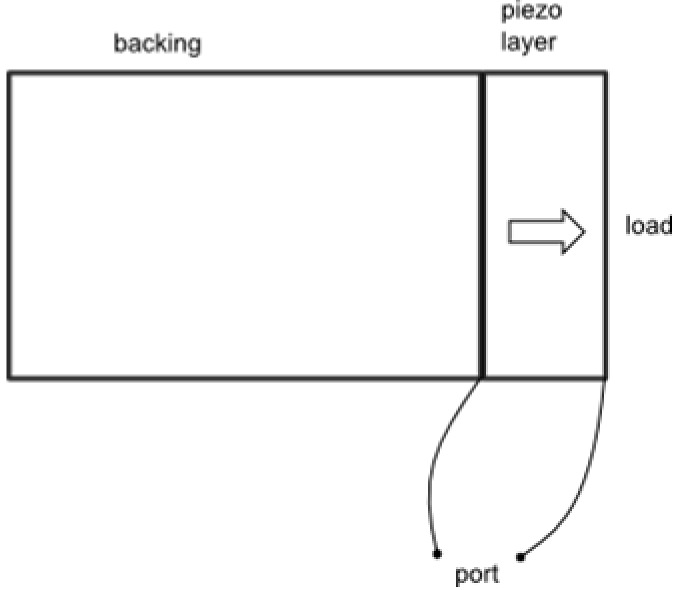
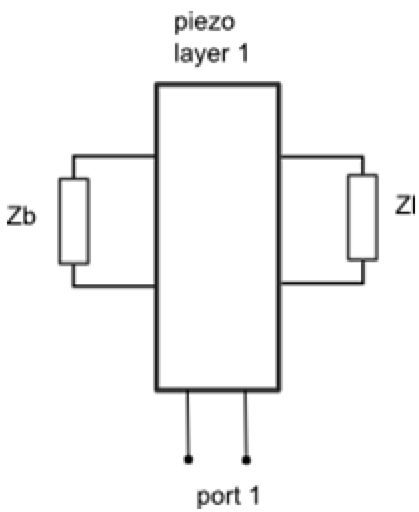
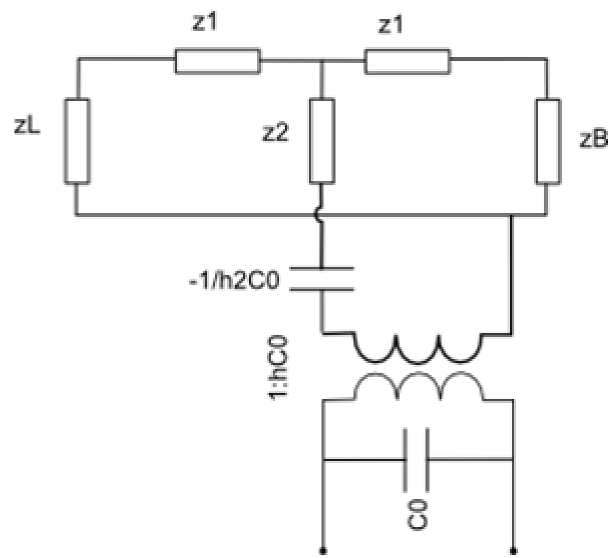
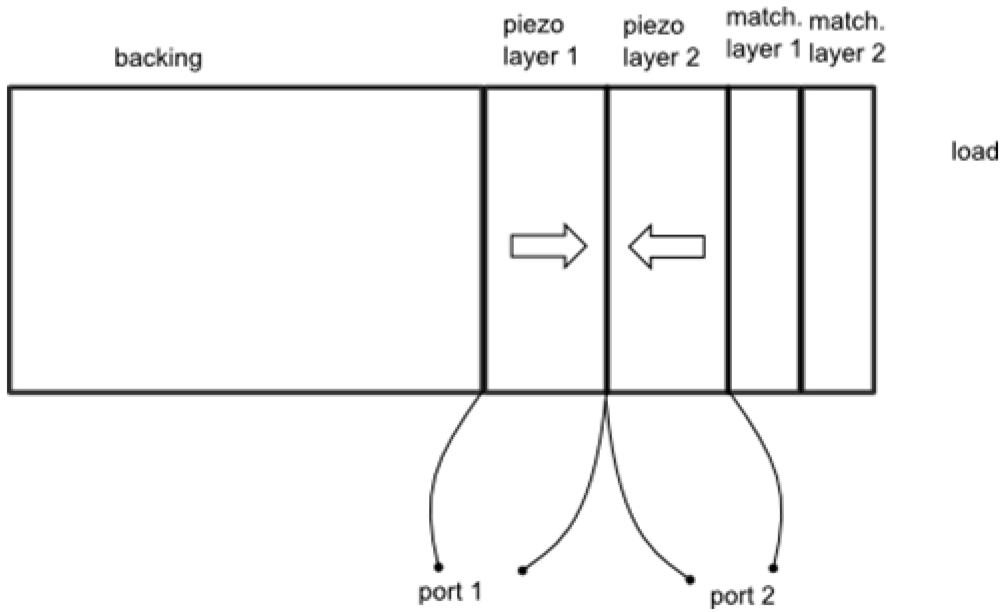
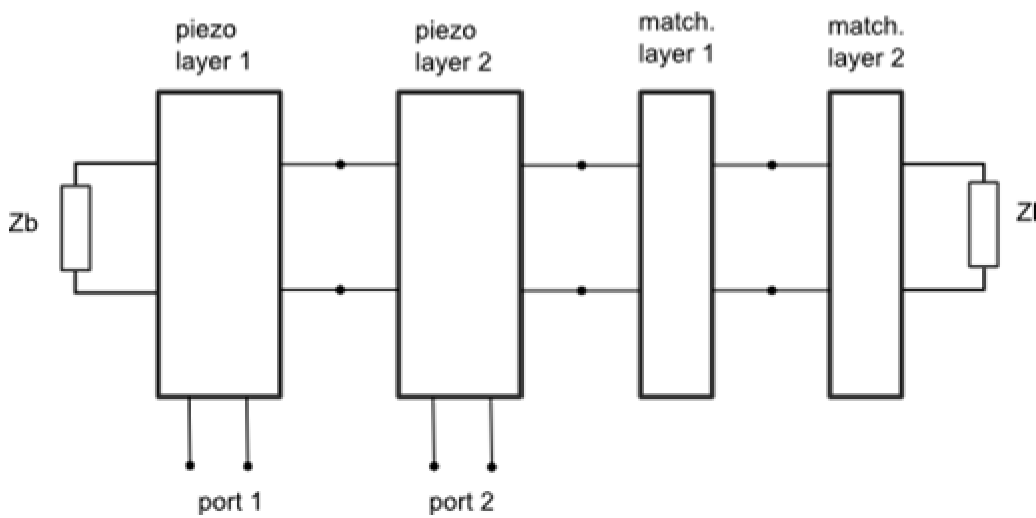

backing
piezo
layer
load
port


piezo
layer 1
Zb
Zl
port 1


z1
z1
zL
z2
zB
-1/h2C0
1:hC0
C0





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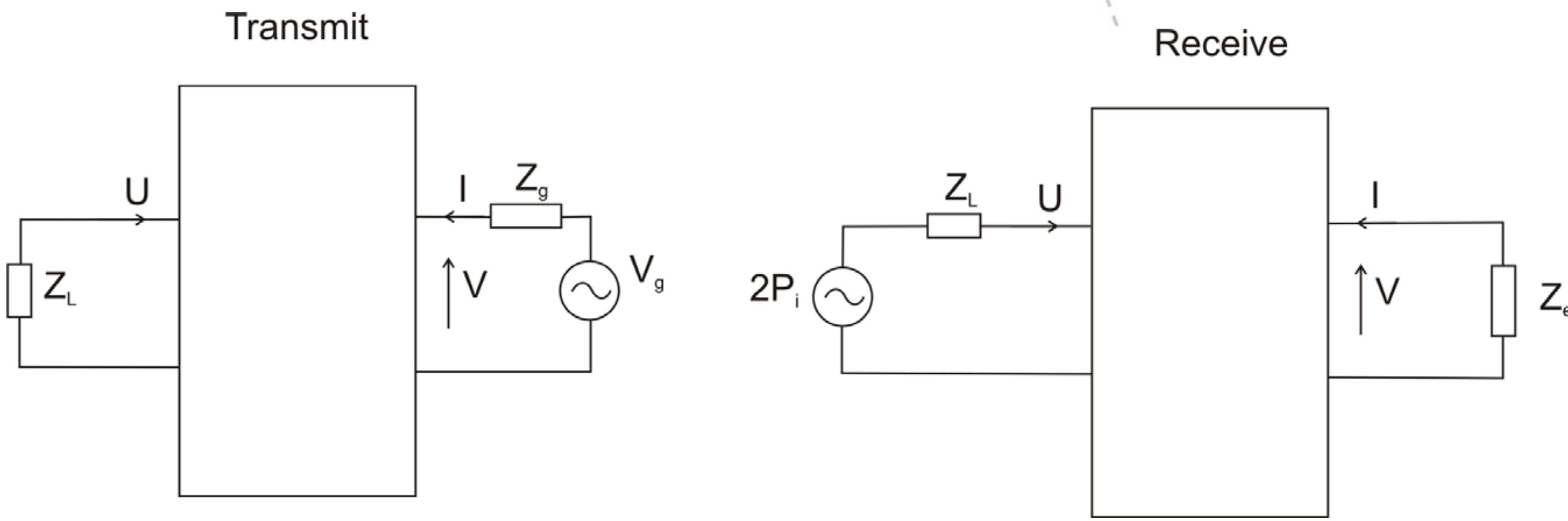
Admittance matrix model (xTrans)

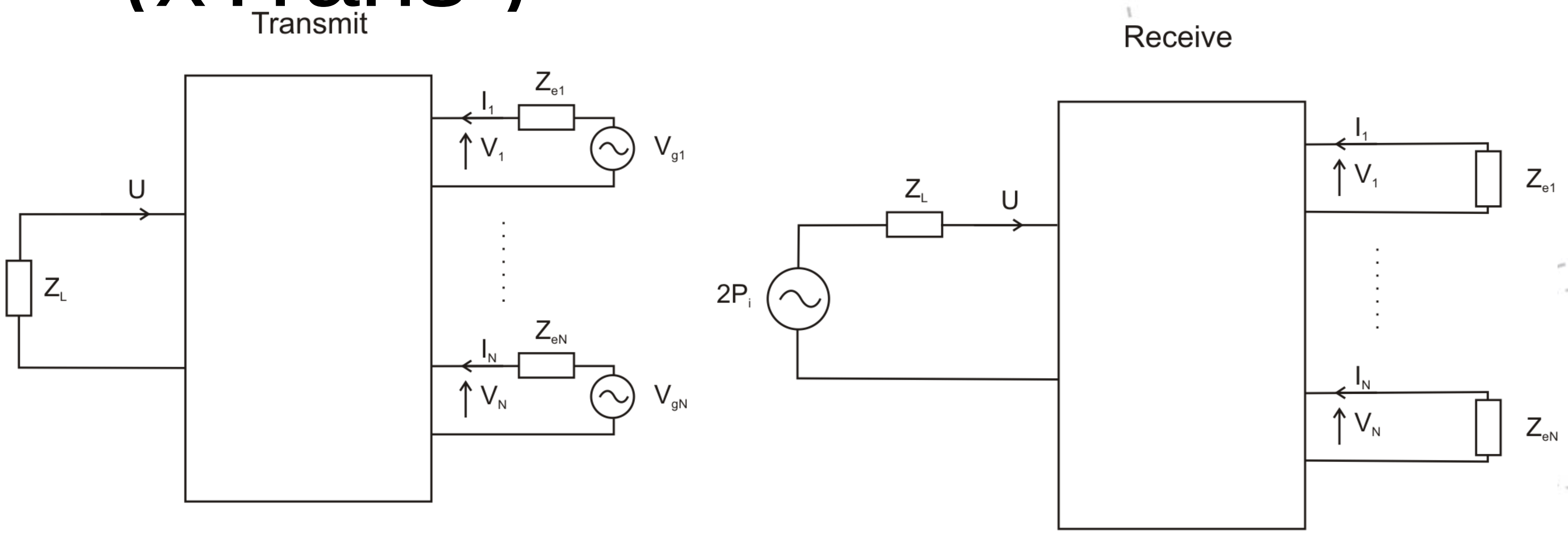


$$\begin{pmatrix} I \\ U \end{pmatrix} = \begin{pmatrix} Y_e & H_{tt} \\ H_{tt} & Y_m \end{pmatrix} \begin{pmatrix} V \\ 2P_i \end{pmatrix}$$

$$H_{tt} = \frac{U}{V_{tt}}$$

$$H_{tt} = \frac{I_r}{2P_i}$$

Admittance matrix model (xTrans)



$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \\ U \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} & H_{tt,s,1} \\ \vdots & \ddots & \vdots & \vdots \\ Y_{N1} & \cdots & Y_{NN} & H_{tt,s,N} \\ H_{tt,s,1} & \cdots & H_{tt,s,N} & Y_m \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \\ 2P_i \end{bmatrix}$$

$$H_{tt,s,n} = \frac{U}{V_n} \quad , \quad V_i = 0, i \neq n$$

$$H_{tt,s,n} = \frac{I_n}{2P_i} \quad , \quad V_i = 0, i \in [1, N]$$

$$\begin{pmatrix} \mathbf{I} \\ U \end{pmatrix} = \begin{pmatrix} \mathbf{Y} & \mathbf{H_{tt}} \\ \mathbf{H_{tt}^T} & Y_M \end{pmatrix} \begin{pmatrix} \mathbf{V} \\ 2P_i \end{pmatrix}$$

$$H_{tt,s,n} = \frac{U}{V_n} \quad , \quad V_j = 0, \; j \neq n$$

$$H_{tt,s,n} = \frac{I_n}{2P_i} \quad , \quad V_j = 0, \; j \in [1, N]$$

$$Y_M = \frac{1}{Z_L + Z_{sM}}$$

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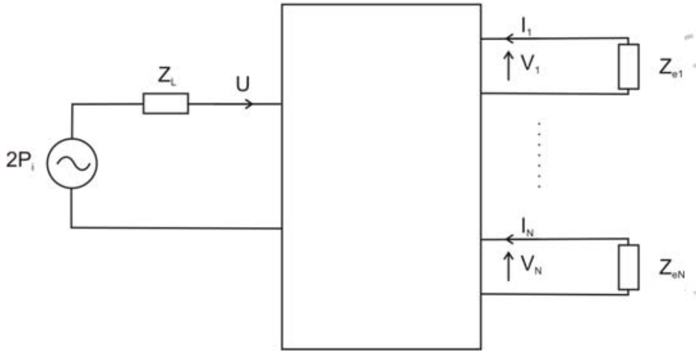
Reflection coefficients

$$\begin{pmatrix} \mathbf{I} \\ U \end{pmatrix} = \begin{pmatrix} \mathbf{Y} & \mathbf{H_{tt}} \\ \mathbf{H_{tt}^T} & Y_M \end{pmatrix} \begin{pmatrix} \mathbf{V} \\ 2P_i \end{pmatrix}$$

Receive



$$R_s = \frac{P_r}{P_i} = 1 - 2Y_M Z_L$$

$$\mathbf{I} = -\mathbf{Y_r V}$$

$$-\mathbf{Y_r V} = \mathbf{YV} + \mathbf{H_{tt}} 2P_i$$

$$\mathbf{V} = -(\mathbf{Y} + \mathbf{Y_r})^{-1} \mathbf{H_{tt}} 2P_i$$

$$U = \left(-\mathbf{H_{tt}^T} (\mathbf{Y} + \mathbf{Y_r})^{-1} \mathbf{H_{tt}} + Y_M\right) 2P_i$$

$$U_r = U - U_i = \left(2Y_M - 2\mathbf{H_{tt}^T} (\mathbf{Y} + \mathbf{Y_r})^{-1} \mathbf{H_{tt}} - \frac{1}{Z_L}\right) P_i$$

$$R_z = \left(1 - 2Y_M Z_L + 2Z_L \mathbf{H_{tt}^T} (\mathbf{Y} + \mathbf{Y_r})^{-1} \mathbf{H_{tt}}\right)$$

$$R_z = \left(R_s + 2Z_L \mathbf{H_{tt}^T} (\mathbf{Y} + \mathbf{Y_r})^{-1} \mathbf{H_{tt}}\right)$$

$$R_o = \left(R_s + 2Z_L \mathbf{H_{tt}^T} \mathbf{Y}^{-1} \mathbf{H_{tt}}\right)$$

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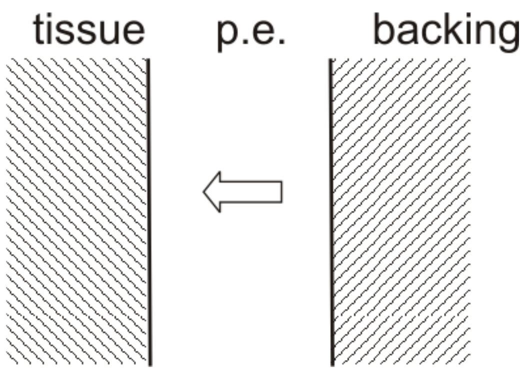
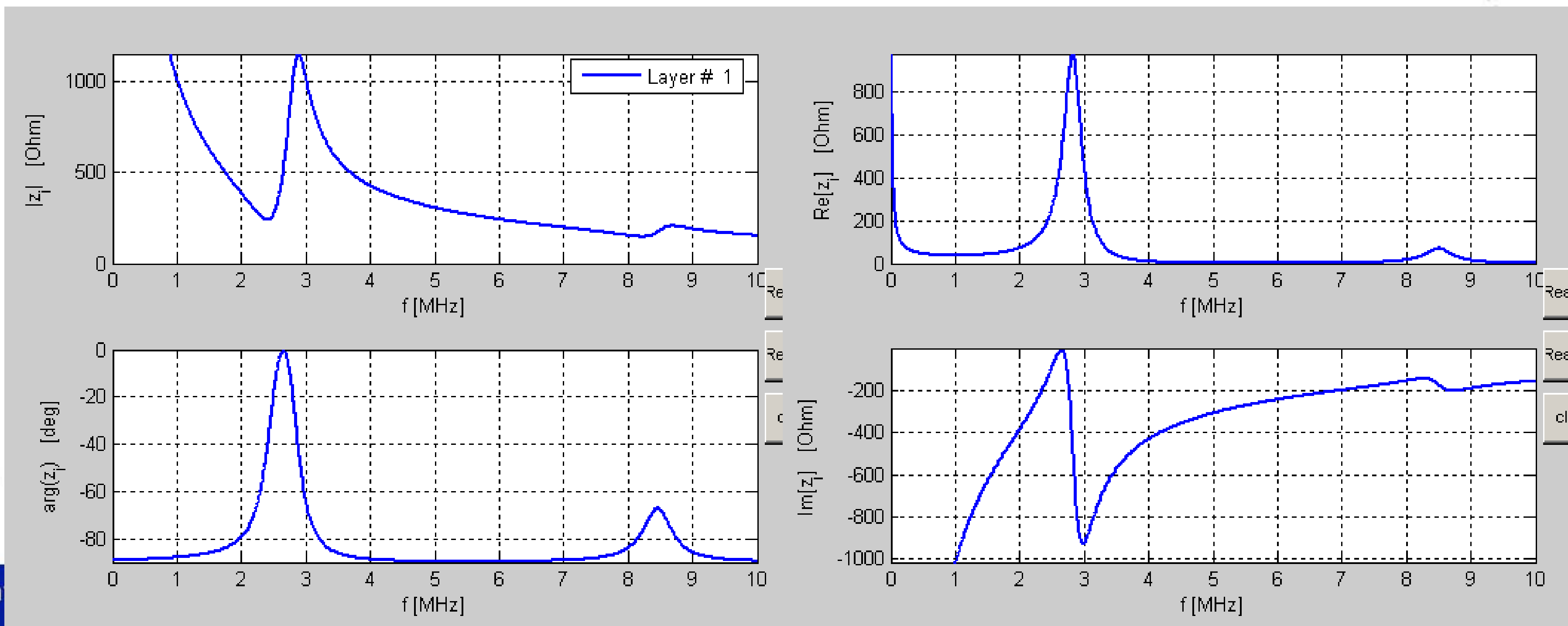
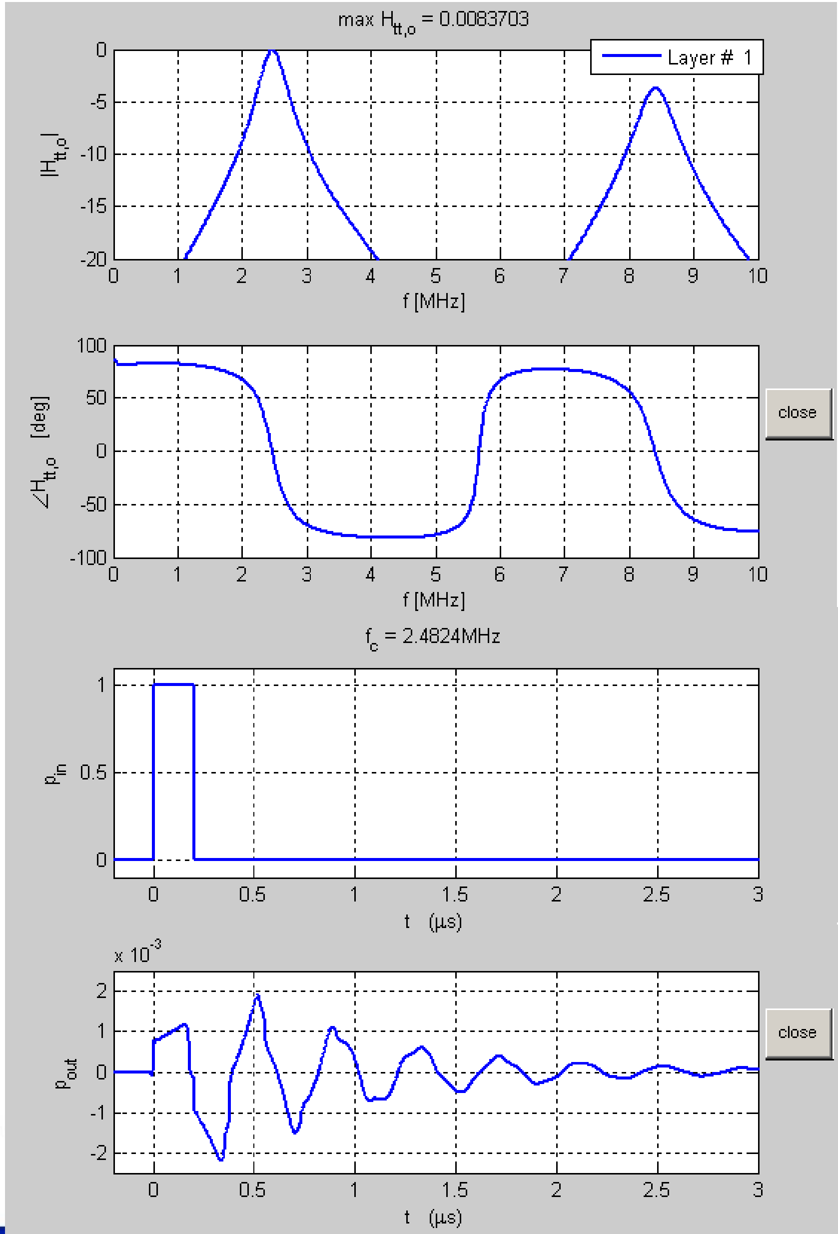
Ex. Pz29 no matching



	h 10^8 V/m	$\varepsilon/\varepsilon_0$	Z MRayl	c m/s	L mm
backing			3		inf
p.e.	19.6	1220	33.6	4440	0.78
tissue			1.65		inf



Ex. Pz29 no matching, cont.



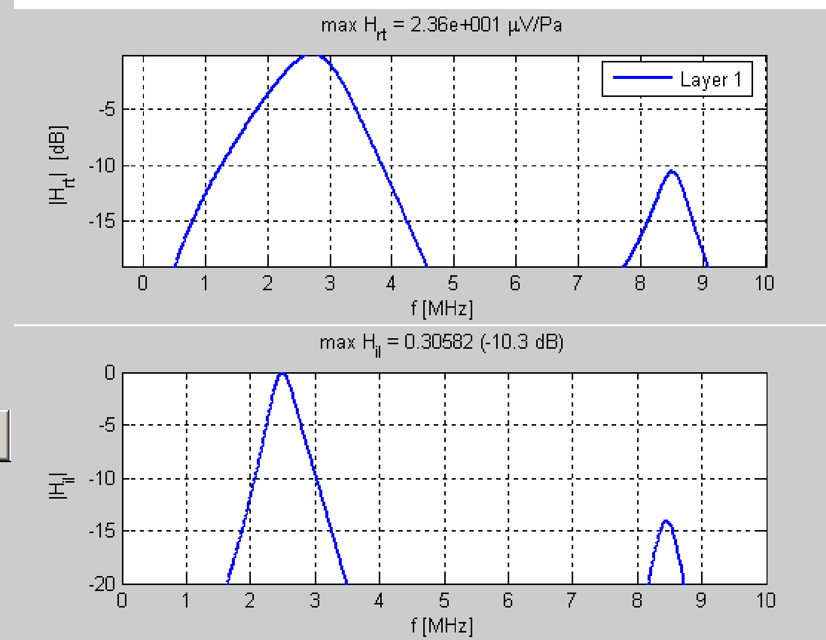
$Bw_{tx,-3dB}$ = 16%

$Bw_{rx,-3dB}$ = 44%

$Bw_{2w,-6dB}$ = 24%

receive load:

500 Ω – 20nH



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